FLOW OF DILUTE SUSPENSIONS OF DEFORMABLE ELLIPSOIDAL PARTICLES IN A PLANE CHANNEL

Kwak Van Dong, Z. V. Pasechnik,
and Yu. I. Shmakov

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The influence of the internal properties of deformable suspended particles on the flow of suspensions in plane channels is investigated.

Rheological equations of state of dilute suspensions of deformable particles, modeled by an ellipsoid of revolution possessing viscosity and elasticity, were obtained in [1] under the assumption that the dispersion medium is a Newtonian liquid while the particles are small, and, by virtue of this, subject to Brownian motion. The peculiarities of the rheological behavior of such suspensions in simple shear flow were investigated in [2].

The peculiarities of the established laminar flow of a suspension [1] in a plane channel are investigated below.

The variability of the shear velocity and the presence of solid boundaries in the flow can result in a number of effects complicating the investigation: migration of the suspended particles, the appearance of secondary flows, disruption of attachment conditions, and a change in the form of the rheological equations of state near the wall are possible.

Migration of suspended particles in the flow will be absent if the inequality [3, 4]

$$\left(\frac{2r_{\text{ef}}}{h}\right)^2\cdot\frac{r_{\text{ef}}v}{\nu}<10^{-6}$$

is satisfied.

The analysis of particle motion near a wall carried out in [5] showed that the corrections which need to be introduced into the rheological equations of state of a suspension near a wall and the slip velocity have the order $O(r_{\text{ef}}/h)$, and therefore they will be omitted in the future and the attachment condition will be adopted at a solid wall as in classical fluid mechanics. As for secondary flows, they are absent in the flow of dilute suspensions in plane channels, as shown in [6].

Thus, if the suspended particles are small enough, the flow of a suspension in a channel will be quasi-one-dimensional (laminar):

$$v_x=0,\ v_y=v(x),\ v_z=0. \tag{1}$$

Since the maximum size of a suspended particle is considerably less than the minimum radius of curvature of the velocity profile v(x), at each point of the stream a particle behaves as in simple shear flow with a shear velocity $K = |dv/dx|$, which is confirmed experimentally [7]. This permits the use of the dependence of the components of the stress tensor on the shear velocity, established in [1] (for simple shear flow), to investigate the laminar flow of a suspension in a plane channel. They contain terms which must be averaged using the distribution function for the angular positions and lengths of the axis of symmetry of a particle, and the main difficulties in the analysis of concrete flows consist in finding this function satisfying a very complicated diffusion equation [1, 2]. If the particle deformations due to hydrodynamic forces are small compared with the average Brownian deformations, the averaging can be carried out first over the ensemble of angular positions of a suspended particle, assumed to be rigid, and then over the ensemble of possible elongations of the suspended particle [8].

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The components of the stress tensor for a suspension, after averaging over the ensemble of angular positions of the particle using the distribution function defined in [9], take the form [2]

$$
\begin{aligned}
t_{xx} &= -p + \frac{2\pi}{5}\left\langle \mu_1 \sum_{j=0}^{\infty} \lambda^j \left(4a_{11,j} - \frac{1}{3} a_{10,j}\right)\right\rangle + \\
&+ \left\{8\pi \left\langle \mu_2 \sum_{j=0}^{\infty} \lambda^j \left(\frac{3}{35} b_{11,j} + \frac{4}{3} b_{22,j}\right)\right\rangle + \frac{16\pi}{5}\left\langle \mu_3 \sum_{j=0}^{\infty} \lambda^j b_{11,j}\right\rangle\right\} K, \\
t_{yy} &= -p - \frac{8\pi}{5}\left\langle \mu_1 \sum_{j=0}^{\infty} \lambda^j \left(\frac{1}{12} a_{10,j} + a_{11,j}\right)\right\rangle + \\
&+ \left\{8\pi \left\langle \mu_2 \sum_{j=0}^{\infty} \lambda^j \left(\frac{3}{35} b_{11,j} - \frac{4}{3} b_{22,j}\right)\right\rangle + \frac{16\pi}{5}\left\langle \mu_3 \sum_{j=0}^{\infty} \lambda^j b_{11,j}\right\rangle\right\} K, \\
t_{zz} &= -p + \frac{4\pi}{15}\left\langle \mu_1 \sum_{j=0}^{\infty} \lambda^j a_{10,j}\right\rangle + \frac{8\pi}{35}\left\langle \mu_2 \sum_{j=0}^{\infty} \lambda^j b_{11,j}\right\rangle K, \\
t_{xy} &= \langle \mu_0 \rangle K + \frac{8\pi}{5}\left\langle \mu_1 \sum_{j=0}^{\infty} \lambda^j b_{11,j}\right\rangle + \left\{\frac{1}{15}\left\langle \mu_2 \left[1 + 4\pi \sum_{j=0}^{\infty} \lambda^j \left(-\frac{1}{7} a_{10,j} + \frac{1}{42} a_{20,j} - 40 a_{22,j}\right)\right]\right\rangle + \right. \\
&\left. + \frac{2}{3}\left\langle \mu_3 \left(1 - \frac{2\pi}{5} \sum_{j=0}^{\infty} \lambda^j a_{10,j}\right)\right\rangle\right\} K, \\
& t_{yx} = t_{xy},\ t_{xz} = t_{zx} = 0,\ t_{yz} = t_{zy} = 0,
\end{aligned}
\tag{2}
$$

where

$$
\frac{\mu_0 - \mu}{\mu\Phi} = \frac{1}{ab^4 a_0'}; \quad \frac{\mu_1}{\mu\Phi \langle D_r \rangle} = \langle F_r \rangle \left\{3\lambda + \frac{\mu/\eta}{2\mu/\eta + 3ab^2\beta_0''} \times \right.
$$

$$
\left. \times \left[\frac{16\pi}{3}\left(\frac{q}{q_0}\right)^{\frac{2}{3}} \left(1 - \frac{q_0}{q}\right)\overline{G} + \frac{9q}{2\mu/\eta + 3ab^2\beta_0''} \frac{d}{dq}(ab^2\beta_0'') - 6\right]\right\};
$$

$$
\frac{\mu_2}{\mu\Phi} = 2\left[\frac{ab^2 a_0''}{ab^4\alpha_0' ab^2\beta_0''} + \frac{1}{ab^4\alpha_0''} - \frac{4}{ab^4\beta_0'(q^2+1)} - \frac{2\mu/\eta}{(2\mu/\eta + 3ab^2\beta_0'')\, ab^2\beta_0''}\right];
$$

$$
\frac{\mu_3}{\mu\Phi} = \frac{2}{(q^2+1)\, ab^4\beta_0'} - \frac{1}{ab^4\alpha_0'}; \quad \overline{G} = \frac{G r_{\text{ef}}^3}{kT};
$$

$$
r_{\text{ef}} = \sqrt[3]{ab^2}; \quad \lambda = \frac{q^2 - 1}{q^2 + 1}; \quad q = \frac{a}{b}; \quad q_0 = \frac{a_0}{b_0}; \quad \sigma = \frac{K}{\langle D_r \rangle};
$$

$$
D_r = \frac{kT}{f_r}; \quad f_r = \frac{4}{3}\pi r_{\text{ef}}^3 \mu F_r; \quad F_r = \frac{4(q^2+1)}{ab^2\alpha_0 q + ab^2\beta_0};
$$

$$
ab^2\alpha_0,\ ab^2\beta_0,\ ab^4\alpha_0',\ ab^4\beta_0',\ ab^2\alpha_0'',\ ab^2\beta_0''
$$

are functions of q defined in [10]; $a_{nm,j}$ and $b_{nm,j}$ are coefficients found from a system of recurrent equations as functions of the parameters q and σ [9]; the symbol < > denotes averaging over the possible elongations of the particle's axis of symmetry using a distribution function F satisfying the equation

$$
-kT\lambda_5 n^2 \frac{d^2F}{dn^2} + \left(\lambda_1 n - 4kTn\lambda_5 - kTn^2 \frac{d\lambda_5}{dn}\right)\frac{dF}{dn} + \left(3\lambda_1 + n\frac{d\lambda_1}{dn}\right)F = 0 \tag{3}
$$

with the boundary conditions

$$
F(n) \to 0 \ \text{as} \ n \to 0, \quad F(n) \to 0 \ \text{as} \ n \to \infty \tag{4}
$$

and the normalization condition. Here n is the length of the semiaxis of symmetry of the ellipsoidal particle;

$$\lambda_1 = -\frac{2ab^2\beta_0''Ga/a_0(1-q_0/q)}{\mu(2+3ab^2\beta_0''\eta/\mu)}; \quad \lambda_5 = \frac{3\beta_0''}{4\pi\mu(2+3ab^2\beta_0''\eta/\mu)}.$$

A solution of Eq. (3) satisfying the conditions (4) was obtained in [2] and has the form

$$F = A\exp\left\{-\frac{8\pi}{3}\overline{G}\left[\left(\frac{q}{q_0}\right)^{2/3} + 2\left(\frac{q_0}{q}\right)^{1/3}\right]\right\};$$

A is determined from the normalization condition.

We represent the components of the stress tensor in the form

$$t_{xx} = -p + t_{xx}^*, \; t_{yy} = -p + t_{yy}^*, \; t_{zz} = -p + t_{zz}^*, \tag{5}$$

and then on the basis of the stress-dynamic equations and (5) we obtain the equations

$$\frac{dt_{xx}^*}{dx} = \frac{\partial p}{\partial x}, \quad \frac{dt_{xy}}{dx} = \frac{\partial p}{\partial y}. \tag{6}$$

It follows from (6) that the pressure drop along the channel is constant, $\partial p/\partial y$ = const, but, in contrast to a Newtonian liquid, the pressure varies across the channel, $\partial p/\partial x \neq 0$. We introduce the apparent viscosity of the suspension,

$$\mu_a = \frac{t_{xy}}{K} = \mu\left[1 + \Phi S_{xy}\left(q_0, \frac{\eta}{\mu}, \overline{G}, \sigma\right)\right], \tag{7}$$

where S_{xy} is a function, known from the solution of the problem of simple shear flow of a suspension, in which the constant shear velocity K is replaced by a variable shear velocity along the channel, $|dv/dx|$.

Integrating Eqs. (6), we obtain

$$p(x, y) - p(0, y) = t_{xx}^*(x), \tag{8}$$

$$\mu_a\frac{dv}{dx} = \frac{\partial p}{\partial y}x + C, \tag{9}$$

where C is the integration constant. From the condition of symmetry of the velocity profile relative to the channel axis we get C = 0.

Changing to the dimensionless variables $\overline{x} = x/h$ and $\overline{v} = v\mu/(h^2|\partial p/\partial y|)$ and using the obvious relation

$$\overline{v}(\sigma) - \overline{v}(0) = \int_0^\sigma \frac{d\overline{v}}{d\overline{x}}\,d\overline{x}, \tag{10}$$

we obtain the solution of this problem in the parametric form

$$\overline{x} = \frac{\langle D_r \rangle}{K^*}(1 + \Phi S_{xy})\sigma, \tag{11}$$

$$\overline{v}(0) - \overline{v}(\sigma) = \left(\frac{\langle D_r \rangle}{K^*}\right)^2\left[\frac{\sigma^2}{2} + \Phi\left(S_{xy}\sigma^2 - \int_0^\sigma S_{xy}\sigma\,d\sigma\right)\right], \tag{12}$$

where

$$K^* = \frac{h}{\mu}\left|\frac{\partial p}{\partial y}\right|.$$

To find $\overline{v}(0)$ we use the boundary condition $\overline{v}|_{\overline{x}=0.5} = 0$ at the wall.

Eliminating the parameter σ from Eqs. (11) and (12), we obtain a velocity diagram $\overline{v}(\overline{x})$; the pressure distribution is given by Eq. (8).

In Fig. 1 we present the results of calculations of the functions $\overline{v}(\overline{x})$ and $P(\overline{x}) = [p(\overline{x}, y) - p(0, y)]/(\Phi h|\partial p/\partial y|)$ for aqueous suspensions of deformable particles with the following parameters: h = 0.005 m; $\partial p/\partial y = -7.12$ N/m²; T = 300°K; $r_{ef} = 10^{-7}$ m; $q_0 = 25$; $\Phi = 0.01$; η/μ = 10,100 $\overline{G}$ = 1000. The results of calculations made for the case of a dispersion medium

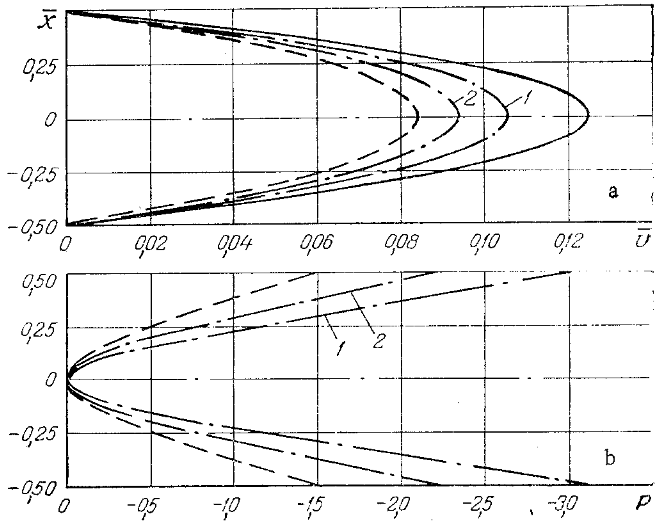

Fig. 1. Distribution of velocity (a) and pressure (b).

and a suspension of rigid particles for the same values of the parameters r_{ef}, q, and Φ are also given here for comparison. The solid line corresponds to the flow of the dispersion medium, the dashed line to that of a suspension of rigid particles, and the dash-dot lines to that of a suspension of deformable particles: 1) $\eta/\mu = 10$; 2) 100.

As follows from Fig. 1a, a decrease in the internal viscosity of the particle material results in an increase in the flow rate of suspension through a channel cross section (for $\eta/\mu = 100$ the flow rate increases by 12% compared with the flow rate of a suspension of rigid particles, and for $\eta/\mu = 10$ it increases by 26%).

It follows from Fig. 1b that a decrease in the parameter η/μ, which characterizes the viscoelastic properties of the particle material, results in enhancement of the viscoelastic behavior of the suspension.

NOTATION

h, channel width; x, y, transverse and longitudinal coordinates; a_0, b_0, a, b, semiaxis of revolution and equatorial radius of the ellipsoidal particle in the undeformed and deformed states; r_{ef}, effective particle radius; Φ, volumetric concentration of suspended particles in the suspension; μ, ν, coefficients of dynamic and kinematic viscosity of the dispersion medium; v_x, v_y, v_z, velocity components; K, shear velocity; t_{ij}, stress tensor; p, pressure in the suspension; η and G, coefficient of dynamic viscosity and shear modulus of the particle material; k, Boltzmann constant; T, absolute temperature; D_r and f_r, coefficients of rotational diffusion and friction of a particle.

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KINETICS OF THE AGGREGATION OF A DILUTE, FINELY DISPERSE SYSTEM AT LOW SHEAR VELOCITIES

Yu. A. Buevich and K. S. Kuvalkin

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The formation of doublets of spherical particles in a shear stream due to Brownian motion is considered with allowance for their hydrodynamic interaction and the breakup of doublets with a low binding energy.

The rheological, thermo-, electro-, and magnetophysical properties of colloidal and other disperse systems depend very strongly on the processes of reversible and irreversible structure formation taking place in the system. Therefore, a theoretical investigation of such properties is impossible without a preliminary physical analysis both of the peculiarities of the occurrence of these processes under various conditions and of their influence on the observable properties of the system. The most important stage in the structure formation of a disperse system is the initial stage of its aggregation, i.e., the formation of doublets from single particles (singlets). Under certain conditions, doublets can subsequently grow through the capture of new singlets, up to the formation of small chains or round aggregates containing a large number of particles, which can take part as certain elementary units in the construction of more complicated branched structures. In dilute systems, as well as in systems with sufficiently weak interaction between particles, the formation just of doublets comprises the main observable form of structure formation.

Brownian coagulation (or flocculation) is usually investigated on the basis of Smolukhovskii's classical concepts. In doing this, the following factors are ignored or not correctly taken into account in the majority of reports: 1) the hydrodynamic interaction between converging particles and the resulting decrease in the effective coefficient of relative Brownian diffusion, 2) the finiteness of the interparticle binding energy and the possibility of the breakup of doublets, and 3) the influence of the "macroscopic" (mean) motion of the system. Attempts to allow for the first factor were made in [1, 2], for the second in [3, 4], and for the third in [5]. All three factors are considered below on the example of a dilute, finely disperse system of single spherical particles having a central interaction potential and suspended in an incompressible liquid entrained in shear flow.

The kinetics of the initial coagulation stage is determined by the velocity of convective interdiffusion of individual pairs of particles. Placing the origin of coordinates at the center of one of the particles, we write the Liouville equation controlling the evolution of the probability density $p(t, \mathbf{r})$ of finding the center of the second particle of a given pair near the point $\mathbf{r}$ at the time t,

$$\partial p/\partial t + \nabla\cdot(p\mathbf{V}) = 0, \quad (1)$$

where the effective relative velocity of the centers of the particles can be represented in the form

$$\mathbf{V} = (\mathbf{b}_{11} + \mathbf{b}_{22} - \mathbf{b}_{12} - \mathbf{b}_{21})\cdot\mathbf{F}. \quad (2)$$

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